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GRAVITATIONAL COLLAPSE OF PERFECT FLUID IN SELF-SIMILAR HIGHER DIMENSIONAL SPACE-TIMES

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We investigate the occurrence and nature of naked singularities in the gravitational collapse of an adiabatic perfect fluid in self-similar higher dimensional space-times. It is shown that strong curvature naked singularities could occur if the weak energy condition holds. Its implication for cosmic censorship conjecture is discussed. Known results of analogous studies in four dimensions can be recovered.

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1. Introduction

The cosmic censorship conjecture, which is widely believed to be true, put forward by Penrose [1] thirty years ago has become the corner stone of general relativity. Moreover, it is being envisaged as a basic principle of nature. The conjecture (in its weak form) asserts that no physically reasonable matter field can give rise to singularities which are observable from asymptotic regions of the space-time. According to the strong version of the conjecture, such singularities are invisible to all observers, i.e., there occurs no naked singularities for any observer, even someone close to it.

However, despite the flurry of activity over the years, the validity of this conjecture is still an open question and very far from being settled. In view of this, it is certainly worthwhile exercise to search for examples which appear to violate the conjecture (for recent reviews and reference on conjecture, see [2,3]). There are of course number of solutions of Einstein's equations, in , which admit naked singularities (see, e.g., [2] and reference therein). This happens in the gravitational collapse of different geometrical configuration and various types of matter. Those received most attention, and which can arise from regular initial data, are generated in self-similar collapse [4,5,6,7,8,9,10]. The self-similar models may be refer to highly symmetric and specialized situation. But the self-similarity is interesting to study

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as it reduces Einstein's field equations to a system of ordinary differential equations and it may simplify physical interpretation of models.

In recent years, superstring and field theories provoked great interest among theoretical physicists in studying physics of higher dimensions [11] and hence significant efforts have been expended to study gravitational collapse models in higher dimensions (HD) [12] but only a few, for example see [13], have studied these problems from viewpoint of the cosmic censorship conjecture. It is shown that a naked singularity forms in the HD spherical collapse of null radiation [13] and of dust as well [14]. However, these models neglect pressure which is likely to play important role in final outcome of the collapse. Ori and Piran [4], in 4D case, have studied numerically self-similar collapse of an adiabatic perfect fluid and have shown that with a soft equation of state the collapse can give rise to a naked singularity [5]. This was followed by analytical discussions of spherical collapse of perfect fluid under assumption of self-similarity [8]. It is therefore interesting to study gravitational collapse of perfect fluid in HD space-times. There is some literature on HD perfect fluid collapse [15] from the viewpoint of the conjecture. However, these studies are restricted to 5D space-time and cannot be reduced to the 4D case.

The main purpose of this paper is to generalize analytical studies, of self-similar collapse of adiabatic perfect fluid collapse, in 4D to $D = n + 2$-dimensional space-times (where $n \geq 2$) and to show that previous results of gravitational collapse, obtained in 4D space-time, are also valid in HD space-time. Infact, It is shown that gravitational collapse of HD space-times gives rise to shell focusing globally naked strong curvature singularities. This is done in section IV. In section II we write the effective Einstein's equations in HD and generalize relevant Cahill and Taub [16] solutions. We have also done matching of these solutions to the HD Vaidya solutions in sections III and, we conclude with a discussion.

2. HD perfect fluid space-times

To facilitate the discussion and set notation, we start with HD self-similar spherically symmetric space-times, which in comoving coordinates reads:

$$ds^2 = e^{\vartheta} dt^2 - e^{\chi} dr^2 - r^2 S^2 d\Omega^2 \tag{1}$$

where ϑ, χ and S are function of r and t and

$$d\Omega^2 = \sum_{i=1}^{n} \left[\prod_{j=1}^{i-1} \sin^2\theta_j \right] d\theta_i^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2 + \ \dots \ + \sin^2\theta_1 \sin^2\theta_2 \dots \sin^2\theta_{n-1} d\theta_n^2 \tag{2}$$

is the metric on an n-sphere and $n = D - 2$, where D is the total number of dimensions. For a perfect fluid, the Einstein equations are

$$R^{ab} - \frac{1}{2} R g^{ab} = (\rho + P) u^a u^b - P g^{ab} \tag{3}$$

where μ is energy density, P is pressure, $u_a = e^{-\vartheta/2}\delta_a^t$ is comoving n-velocity, and we choose units with $8\pi G = c = 1$. A spherically symmetric self-similar solution is one in which the space-time admits a homothetic Killing vector ξ which satisfies

$$\mathcal{L}_\xi g_{ab} = \xi_{a;b} + \xi_{b;a} = 2g_{ab} \tag{4}$$

where $\mathcal{L}$ denotes the Lie derivative. This means that by a suitable transformation of coordinates, all metric coefficients and dependent variables can be put in the form in which they are functions of a single independent variable (say z), which is dimensionless combination of space and time coordinates and hence ϑ, χ and S are functions of the self-similarity variable $z = t/r$.

The simplest equation of state compatible with similarity solution is one of the form $p(z) = \alpha\zeta(z)$, where $0 \leq \alpha \leq 1$ is a constant.

The conservation equations $T^{ab}_{;b} = 0$ are immediately integrated to give

$$e^\vartheta = \gamma(\zeta z^2)^{-2\alpha/(1+\alpha)} \tag{5}$$

$$e^\chi = \eta(\zeta)^{-2/(1+\alpha)} S^{-2n} \tag{6}$$

where γ and η are integration constants. The remaining field equations reduce to set of ordinary differential equations:

$$G^t_t = -e^{-\vartheta}\left[\frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2} + \frac{n}{2}\frac{\dot{S}}{S}\dot{\chi}\right] + e^{-\chi}\Big[\frac{n(n-1)}{2} + n\frac{\ddot{S}}{S}z^2 + \frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2}z^2 - n(n-1)\frac{\dot{S}}{S}z + \frac{n}{2}\dot{\chi}z - \frac{n}{2}\frac{\dot{S}}{S}\dot{\chi}z^2\Big] - \frac{n(n-1)}{2}\frac{1}{S^2} = -\zeta \tag{7}$$

$$G^r_r = -e^{-\vartheta}\left[n\frac{\ddot{S}}{S} - \frac{n}{2}\frac{\dot{S}}{S}\dot{\vartheta} + \frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2}\right] + e^{-\chi}\Big[\frac{n(n-1)}{2} + \frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2}z^2 - n(n-1)\frac{\dot{S}}{S}z + \frac{n}{2}\dot{\vartheta}z + \frac{n}{2}\frac{\dot{S}}{S}\dot{\vartheta}z^2\Big] - \frac{n(n-1)}{2}\frac{1}{S^2} = p \tag{8}$$

$$G^{\theta_1}_{\theta_1} = e^{-\vartheta}\left[\frac{\ddot{\chi}}{2} + \frac{\dot{\chi}^2}{4} + (n-1)\frac{\ddot{S}}{S} - (n-1)(n-2)\frac{\dot{S}^2}{S^2} + \frac{(n-1)}{2}(\dot{\chi}-\dot{\vartheta})\frac{\dot{S}}{S} - \frac{\dot{\chi}\dot{\vartheta}}{4}\right] + e^{-\chi}\Big[-\frac{\ddot{\vartheta}z^2}{2} + \dot{\vartheta}z + \frac{\dot{\vartheta}^2 z^2}{4} + (n-1)\frac{\ddot{S}z^2}{S} - \frac{(n-1)(n-2)}{2}\frac{\dot{S}^2\,z^2}{S^2} - (n-1)(n-2)\frac{\ddot{S}\,z}{S} + \frac{(n-1)}{2}(\dot{\chi}-\dot{\vartheta})\left(1 - \frac{\ddot{S}\,z}{S}\right)z - \dot{\chi}\dot{\vartheta}z^2 + \frac{(n-1)(n-2)}{2}\Big] - \frac{(n-1)(n-2)}{2}\frac{1}{S^2} = p \tag{9}$$

$$G^{\theta_1}_{\theta_1} = G^{\theta_2}_{\theta_2} = \ \ldots\ = G^{\theta_n}_{\theta_n} \tag{10}$$

$$G^t_r = \frac{n}{2}\frac{z}{S}\left[2\ddot{S} - \dot{S}\dot{\vartheta} - \dot{S}\dot{\chi} + \frac{S\dot{\chi}}{z}\right] = 0 \tag{11}$$

where the overdot denote the derivative with respect similarity parameter z. We have assumed the following expressions for pressure and energy

$$P = \frac{p(z)}{r^2}, \qquad \rho = \frac{\zeta(z)}{r^2} \tag{12}$$

Elimination of $\ddot{S}$ from Eqs. (7) and (8) leads to

$$\frac{n(n-1)}{2}\left(\frac{\dot{S}}{S}\right)^2 V + \frac{n}{2}\frac{\dot{S}}{S}\left(\dot{V} + 2nze^{\vartheta}\right) + e^{\chi+\vartheta}\left[-\zeta - \frac{n(n-1)}{2}e^{-\chi} + \frac{n(n-1)}{2}\frac{1}{S^2}\right] = 0 \tag{13}$$

and

$$V\dot{(}z) = \frac{2}{n}ze^{\vartheta}\left[(\zeta+p)e^{\chi} - n\right] = \frac{2}{n}ze^{\vartheta}(\Theta - n) \tag{14}$$

where the quantities V and Θ are defined as

$$V(z) = e^{\chi} - z^2 e^{\vartheta}, \qquad \Theta(z) = (\zeta + p)e^{\chi} \tag{15}$$

One can also write, $\Theta = r^2 e^{\chi}\left(T^0_0 - T^1_1\right)$. Thus the matter satisfy weak energy condition: $\rho \geq 0$ and $\rho + p \geq 0$ [17] if and only if $T_{ab}W^aW^b \geq 0$, for all nonspacelike vector W^a, which implies that $\Theta(z) \geq 0$ for all z.

3. Junction conditions

We consider a spherical surface with its motion described by a time-like $n+1$ surface Σ, which divides space-times into interior and exterior manifolds. We shall first cut the space-times along time like hypersurface, and then join the internal part with the outgoing HD Vaidya solutions. The metric on the whole space-times can be written in the form

$$ds^2 = \begin{cases} e^{\vartheta}dt^2 - e^{\chi}dr^2 - r^2S^2d\Omega^2, & r \leq r_{\Sigma}, \\ \left[1 - \frac{2m(v)}{(n-1)\mathbf{r}^{(n-1)}}\right]dv^2 + 2dvd\mathbf{r} - \mathbf{r}^2d\Omega^2, & r \geq r_{\Sigma}. \end{cases} \tag{16}$$

The metric on the hypersurface $r = r_{\Sigma}$ is given by

$$ds^2 = d\tau^2 - \mathcal{R}^2(\tau)d\Omega^2 \tag{17}$$

For the junction conditions, we suitably modify the approach given in [18,19] for our HD case. Hence we have to demand

$$(ds^2_-)_{\Sigma} = (ds^2_+)_{\Sigma} = (ds^2)_{\Sigma} \tag{18}$$

The second junction condition is obtained by requiring the continuity of the extrinsic curvature of Σ across the boundary. This yields

$$K^-_{ij} = K^+_{ij} \tag{19}$$

where $K^{\pm}_{ij}$ is extrinsic curvature to Σ, given by

$$K^{\pm}_{ij} = -n^{\pm}_{\alpha}\frac{\partial^2 x^{\alpha}_{\pm}}{\partial\xi^i\partial\xi^j} - n^{\pm}_{\alpha}\Gamma^{\alpha}_{\beta\gamma}\frac{\partial x^{\beta}_{\pm}}{\partial\xi^i}\frac{\partial x^{\gamma}_{\pm}}{\partial\xi^j} \tag{20}$$

and where $\Gamma^{\alpha}_{\beta\gamma}$ are Christoffel symbols, $n^{\pm}_{\alpha}$ the unit normal vectors to Σ, x^{α} are the coordinates of the interior and exterior space-time and ξ^{i} are the coordinates that defines Σ. From the junction condition (18), we obtain

$$\frac{dt}{d\tau} = \frac{1}{e^{\vartheta(r_{\Sigma},t)/2}} \tag{21}$$

$$r_{\Sigma} S(r_{\Sigma},t) = \mathbf{r}(\tau) \tag{22}$$

$$\left(\frac{dv}{d\tau}\right)^{-2}_{\Sigma} = \left[1 - \frac{2m(v)}{(n-1)\mathbf{r}^{(n-1)}} + 2\frac{d\mathbf{r}}{dv}\right] \tag{23}$$

The non-vanishing components of intrinsic curvature K_{ij} of Σ can be calculated and the result is

$$K^{-}_{\tau\tau} = \left(-e^{-\chi/2}\vartheta_{r}\right)_{\Sigma} \tag{24}$$

$$K^{-}_{\theta_1\theta_1} = \left[e^{-\chi/2} rS\left(S + rS_{r}\right)\right]_{\Sigma} \tag{25}$$

$$K^{+}_{\tau\tau} = \left[\frac{d^2v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^{n}}\right]_{\Sigma} \tag{26}$$

$$K^{+}_{\theta_1\theta_1} = \left[\mathbf{r}\frac{d\mathbf{r}}{d\tau} + \left(\frac{dv}{d\tau}\right)\left(1 - \frac{2m(v)}{(n-1)\mathbf{r}^{(n-1)}}\right)\mathbf{r}\right]_{\Sigma} \tag{27}$$

$$K^{\pm}_{\theta_i\ \theta_i} = \sin^2\theta K^{\pm}_{\theta_{i-1}\ \theta_{i-1}} \tag{28}$$

where subscripts r and t denote partial derivative with respect to r and t respectively. The unit normal to the Σ are given by

$$n^{-}_{\alpha} = (0, e^{\chi(r_{\Sigma},t)/2}, 0, ..., 0) \tag{29}$$

$$n^{+}_{\alpha} = \left[1 - \frac{2m(v)}{(n-1)\mathbf{r}^{(n-1)}} + 2\frac{d\mathbf{r}}{dv}\right]^{-1/2}\left(-\frac{d\mathbf{r}}{dv}, 1, 0, ..., 0\right) \tag{30}$$

From Eqs. (19), (25) and (27) we have

$$\left[\left(\frac{dv}{d\tau}\right)\left(1 - \frac{2m(v)}{(n-1)\mathbf{r}^{(n-1)}}\right)\mathbf{r} + \mathbf{r}\frac{d\mathbf{r}}{d\tau}\right] = \left[e^{-\chi/2} rS(S + rS_{r})\right]_{\sum} \tag{31}$$

With the help of Eqs. (21), (22) and (23), we can write Eq. (31) as

$$m(v) = \frac{n-1}{2}(rS)^{(n-1)}\left[1 + \frac{r^2 S_t^2}{e^{\vartheta}} - \frac{(S + rS_r)^2}{e^{\chi}}\right] \tag{32}$$

which is the total energy entrapped inside the surface Σ. This expression (32) is HD generalization of well known mass function introduced by Cahill and McVittie [20]. From Eqs. (24) and (26), using (21), we have

$$\left[\frac{d^2v}{d\tau^2}\left(\frac{dv}{d\tau}\right)^{-1} - \left(\frac{dv}{d\tau}\right)\frac{m(v)}{\mathbf{r}^{n}}\right]_{\Sigma} = -\left(\frac{\vartheta_r}{2e^{\chi/2}}\right)_{\Sigma} \tag{33}$$

Substituting Eqs. (21), (22) and (32) into (31) we can write

$$\left(\frac{dv}{d\tau}\right)_{\Sigma} = \left[\frac{S + rS_r}{e^{\chi/2}} + \frac{rS_t}{e^{\vartheta/2}}\right]_{\Sigma}^{-1} \tag{34}$$

Differentiating (34) with respect to τ and using Eqs. (32), we can rewrite (33) as

$$-\left(\frac{\vartheta_r}{2e^{\chi/2}}\right)_{\Sigma} = \Bigg[\Bigg[-\frac{r}{e^{\chi/2}}S_{tr} + \frac{\chi_t(S + rS_r)}{2e^{\chi/2}} + \frac{r\vartheta_t S_t}{2e^{\vartheta/2}} - \frac{rS_{tt}}{e^{\vartheta/2}} - \frac{n-1}{2}\frac{rS_t^2}{e^{\vartheta/2}S} - \frac{S_t}{e^{\chi}} + \frac{(n-1)e^{\vartheta/2}}{rS} \times \left(\frac{(S + rS_r)^2}{e^{\chi}} - 1\right)\Bigg]\left[\frac{S + rS_r}{e^{\chi/2}} + \frac{rS_t}{e^{\vartheta/2}}\right]^{-1}\frac{1}{e^{\vartheta/2}}\Bigg]_{\Sigma} \tag{35}$$

Next we translate the above equation in terms of $z = t/r$, which is given by

$$-e^{-\vartheta}\left[n\frac{\ddot{S}}{S} - \frac{n}{2}\frac{\dot{S}}{S}\dot{\vartheta} + \frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2}\right] + e^{-\chi}\Big[\frac{n(n-1)}{2} + \frac{n(n-1)}{2}\frac{\dot{S}^2}{S^2}z^2 - n(n-1)\frac{\dot{S}}{S}z + \frac{n}{2}\dot{\vartheta}z + \frac{n}{2}\frac{\dot{S}}{S}\dot{\vartheta}z^2\Big] - \frac{n(n-1)}{2}\frac{1}{S^2} = -\frac{n}{2}\frac{ze^{-(\chi+\vartheta)/2}}{S}\left[2\ddot{S} - \dot{S}\dot{\vartheta} - \dot{S}\dot{\chi} + \frac{S\dot{\chi}}{z}\right] \tag{36}$$

Comparing (36) with (8) and (11), we can finally write

$$(P)_{\Sigma} = 0 \tag{37}$$

Eq. (37) shows that the pressure will vanish at the boundary which implies radiation cannot exist and exterior space-time $\mathcal{V}_E$ is HD Schwarzschild space-time.

4. The structure of singularities in HD spherical collapse

Let $K^a = dx^a/dk$ be the tangent vector to the null geodesics, where k is an affine parameter. Then $g_{ab}K^aK^b = 0$ and it follows that along null geodesics, we have $\xi^a K_a = C$, where C is a constant. From this algebraic equation and the null condition, we get the following exact expressions for K^t and K^r:

$$K^t = \frac{C\left[z \pm e^{\chi}\Pi\right]}{r\left[e^{\chi} - e^{\vartheta}z^2\right]} \tag{38}$$

$$K^r = \frac{C\left[1 \pm ze^{\vartheta}\Pi\right]}{r\left[e^{\chi} - e^{\vartheta}z^2\right]} \tag{39}$$

where $\Pi = \sqrt{e^{-\chi-\vartheta}} > 0$. We shall follow mainly arguments of Joshi and Dwivedi [8] in our subsequent analysis. Radial null geodesics, by virtue of Eqs. (38) and (39), satisfy

$$\frac{dt}{dr} = \frac{z \pm e^{\chi}\Pi}{1 \pm ze^{\vartheta}\Pi} \tag{40}$$

At this point, we note that a curvature singularity forms at the origin $r = 0$, where the physical quantities like density and pressure diverges. The singularity is at least locally naked if there exist radial null geodesics emerging from the singularity, and

if no such geodesics exist it is a black hole. If the singularity is naked, then there exists a real and positive value of z_0 as a solution to the algebraic equation [2]

$$z_0 = \lim_{t\to 0\, r\to 0} z = \lim_{t\to 0\, r\to 0} \frac{t}{r} = \lim_{t\to 0\, r\to 0} \frac{dt}{dr} \tag{41}$$

Using (40) and L'Hôpital's rule we can derive the following equation

$$V(z_0)\Pi(z_0) = 0 \tag{42}$$

Since $\Pi > 0$, this implies that

$$V(z_0) = 0 \tag{43}$$

This algebraic equation governs the behavior of the tangent near the singular points. The central shell focusing is at-least locally naked if Eq. (43) admits one or more positive roots (see also Waugh and Lake [6], and Ori and Piran [4]). The values of the roots give the tangents of the escaping geodesics near the singularity. Thus the visibility of the singularity depends on the existence of positive roots to Eq. (43).

Next, we ask the question when this will be realized in terms of the parameter in self-similar field equations. This can be achieved by further analysis of the self-similar field equations. To this end, we define two new functions $y = z^{\beta}$ and $U^2 = e^{\chi-\vartheta}/z^2 = y^{-n}\zeta^{-n\beta}S^{-2n}$. Here $0 \le \alpha \le 1$, $\delta = 1+\alpha$ and $\beta = 2(1-\alpha)/n(1+\alpha)$

With this transformations Eq.(13) and (14), respectively, become

$$\frac{n(n-1)}{2}\left(\frac{S'}{S}\right)^2 \beta^2 y^2 (U^2-1) + \left(\frac{n}{2}\frac{S'}{S}\right)\beta y \left[2(n-1) + \frac{2}{n}\delta y^n \zeta^{n\beta/2} U^2\right]$$
$$- \left[\frac{n(n-1)}{2} + \left(1 - \frac{n(n-1}{2\zeta S^2}\right) y^n \zeta^{n\beta/2} U^2\right] = 0 \tag{44}$$

$$\beta y \frac{\zeta'}{\zeta} = \frac{1}{U^2-\alpha}\left(2\alpha - n\delta\beta y U^2 \frac{S'}{S} - \frac{1}{n}\delta^2 y^n \zeta^{n\beta/2} U^2\right) \tag{45}$$

We analyze the above differential equations near the point $y = y_0 = y(z_0)$ with the condition that $U(y_0) = (\zeta_0 z_0)^{-n\beta} S_0^{-2n} = 1$.

Let us write

$$\zeta(z) = \zeta_0 + \zeta_0 \sum_{k=1}^{\infty} \zeta_k (y-y_0)^k \tag{46}$$

$$S(z) = S_0 + S_0 \sum_{k=1}^{\infty} S_k (y-y_0)^k \tag{47}$$

which gives $\zeta'(y_0) = \zeta_0\zeta_1$ and $S'(y_0) = S_0 S_1$. Eqs. (44) and (45) can be recast as

$$\zeta_1 = \frac{1}{\beta y_0 (1-\alpha)}\left(2\alpha - n\beta\delta y_0 S_1 - \frac{1}{n}\delta^2 y_0^n \zeta_0^{n\beta/2}\right) \tag{48}$$

$$\beta y_0 S_1 = \frac{n}{n(n-1) + \delta y_0^n \zeta_0^{n\beta/2}}\left[\frac{n-1}{2} + \frac{1}{n}\left(1 - \frac{n(n-1)}{2\zeta_0 S_0^2}\right) y_n \zeta_0^{n\beta/2}\right] \tag{49}$$

Eliminating S_1 and S_0 from the above equations leads to

$$Y^{2n} + \left[mw - \frac{n^2(n-1)}{2}w\right] Y^{n+1} + \left[\frac{n(n-1)}{2}\delta - 2n\alpha + n^2\right] \times Y^n + n(n-1)\delta mwY + \frac{n^3(n-1)}{2}\delta^2 - 2n^2(n-1)\alpha\delta = 0 \tag{50}$$

where $Y = \delta^{2/n}\zeta_0^{\beta/2}y_0$, $m = (\beta(1-\alpha)\zeta_1)/\zeta_0^{\beta-1}$ and $w = n\zeta_0^{(\beta-2)/2}/\delta^{2/n}$. This algebraic equation ultimately decides the final fate of the collapse. In general, the existence of real positive roots of above algebraic equation will put a limitation on the physical parameters ζ_0 and ζ_1. Thus the existence of a real positive root of $V(z) = 0$ (and, hence existence of a naked singularity) is characterized by the values of physical parameters ζ_0 and ζ_1. It is easy to check that the above equation can admit at the most four real positive roots for all n. In similar situation of , one gets a quartic equation [8].

A naked singularity can be considered as a physically significant singularity, if it can escape from singularity to far away observers for a finite period of time. The analysis to show this is similar to that of 4D case [8]. Here we shall restrict ourselves to main results and readers are urged to refer original reference for the details of the mathematical description.

The singularities are visible for a finite period of time, if infinity of integral curves escape from the singularity. To see this, we write the equation of geodesics in the from $r = r(z)$. Using Eqs. (38) and (39) we obtain,

$$\frac{dz}{dr} = \frac{V(z)\Pi(z)}{r[1 + ze^{\vartheta}\Pi]} \tag{51}$$

We have demonstrated that a singularity to be naked, $V(z) = 0$ must have at least one real positive root. Let $z = z_0$ be a simple root of $V(z) = 0$. We could then decompose V(z) as

$$V(z) = \frac{2}{n}(z - z_0)z_0 e^{\vartheta(z_0)}[\Theta_0 - n]$$

Where $\Theta_0 = \Theta(z_0)$. On inserting this expression of $V(z)$ in Eq. (51) and integrating, we arrive at

$$r = \Xi(z - z_0)^{n/[\Theta_0 - n]} \tag{52}$$

where Ξ is the integration constant that labels the different geodesic. Thus, when $\Theta_0 > n$, an infinity of integral curves with tangent $z = z_0$ would escape from the singularity. In corresponding 4D case integral curves meet the singularity in past if $\Theta_0 > 2$ [8]. Further, as in 4D case, it can be shown that an infinity of integral curves will escape from singularity if $0 < \Theta_0 < \infty$. It is seen that $\Theta_0 > 0$ holds only if the weak energy condition is true. Thus we have shown that an infinity of integral curves would escape the singularity provided the weak energy condition is fulfilled.

The naked singularities discussed in previous section are locally naked, i.e., invisible to an asymptotic observer. Next, we briefly describe, when geodesics can reach an asymptotic observer. This follows from the analysis of the Eq. (51), which can be rearranged as

$$r = \Xi\, exp\left[\int \frac{[1+ze^{\vartheta}\Pi]}{(z-a_1)\psi(z)}dz - \int \frac{[1+ze^{\vartheta}\Pi]}{(z-a_2)\psi(z)}dz\right] \tag{53}$$

where

$$\psi(z) = \frac{(a_1-a_2)V(z)\Pi(z)}{(z-a_1)(z-a_2)}$$

where a_1 and a_2 (with $a_1 > a_2$) are two consecutive roots of the equation $V(z) = 0$. Thus all integral curves can meet the singularity in past with tangent $z = a_1$, and $r = \infty$ can be realized in future along same integral curves at $z = a_2$. Thus the singularities will be globally naked and an infinity of curves would emanate from the singularity to reach a distant observer.

Finally, we need to determine the curvature strength of naked singularities, which is an important aspect [21]. A singularity is gravitationally strong or simply strong in the sense of Tipler [22] if every volume element is crushed to zero dimensions at the singularity, and weak otherwise (i.e., if it remains finite). It is widely believed that a space-time does not admit an extension through a singularity if it is a strong curvature singularity in the sense of Tipler [22]. A necessary and sufficient measure for a singularity to be strong has been given by Clarke and Królak [23] that for at least one non-spacelike geodesic with affine parameter k, in the limiting approach to the singularity, we must have

$$\lim_{k\to 0} k^2\psi = \lim_{k\to 0} k^2 R_{ab}K^aK^b > 0 \tag{54}$$

where R_{ab} is the Ricci tensor. Our purpose here is to investigate the above condition along future directed radial null geodesics that emanate from the naked singularity. Eq. (54) can be expressed as

$$\lim_{k\to 0} k^2\psi = \lim_{k\to 0} k^2 \frac{(\zeta+p)C^2e^{\vartheta}[z+e^{\chi}\Pi]^2}{r^4[e^{\chi}-z^2e^{\vartheta}]^2} \tag{55}$$

Using Eqs. (38), (39), and L'Hôpital's rule, Eq. (55) turns out to be

$$\lim_{k\to 0} k^2\psi = \frac{n^2\Theta_0}{(\Theta_0+n)^2} > 0 \tag{56}$$

Thus, along radial null geodesics strong curvature condition is satisfied if $\Theta_0 > 0$, which is also a necessary condition for the energy condition. Thus it follows that singularities are gravitationally strong if the weak energy condition is satisfied.

5. Discussion and concluding remarks

General relativity was formulated in space-time with four dimensions, of course. However, there are theoretical hints that we might live in a world with more than . A generalization of general relativity to HD has been of considerable interest in recent times. For further progress toward an understanding of spherical collapse, from the viewpoint of the cosmic censorship, one would like to know the effect of extra dimensions on the existence of a naked singularity. The relevant questions would be, for instance, whether such solutions remain naked with the introduction of extra dimensions. Do they always become covered? Does the nature of the singularity change? Our analysis shows that none of the above hold. Infact, the gravitational collapse of perfect fluid in HD self-similar spherically symmetric space-times lead to strong curvature globally naked singularities.

It is shown that the singularity at $t = 0$, $r = 0$ is atleast locally naked when $V(z_0) = 0$ has a simple positive root and a infinity of future directed integral curves escape from the singularity provided $0 < \Theta_0 < \infty$. Further such a singularity could be globally naked as well provided $V(z_0) = 0$ has atleast two simple positive roots. Further along null rays emanating from the singularity, the strong curvature condition (54) is satisfied. Thus the naked singularities here could be considered as a physically significant singularity, and hence cannot be ignored. Thus, this offers a serious counter-evidence to the conjecture.

As mentioned earlier, the conjecture has yet no proof for either version. However, It appears to be a growing body of evidence against the conjecture. These examples showing the occurrence of naked singularities remain important and may be valuable if one attempts to formulate the notion of the conjecture in precise mathematical form. They may add some insight in issues involved in cosmic censorship. Indeed, the singularity arising in massless scalar field led to a formulation of weak censorship [24]. Thus there may be much to be learned from studying such counter examples.

To sum up, this generalizes the previous studies of self-similar spherical gravitational collapse in to $D = n + 2$ dimensional space-times and when $n = 2$ one recovers the results of analogous study in 4D.

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